A Few Problems to Motivate Numerical Libraries for Grid Computations

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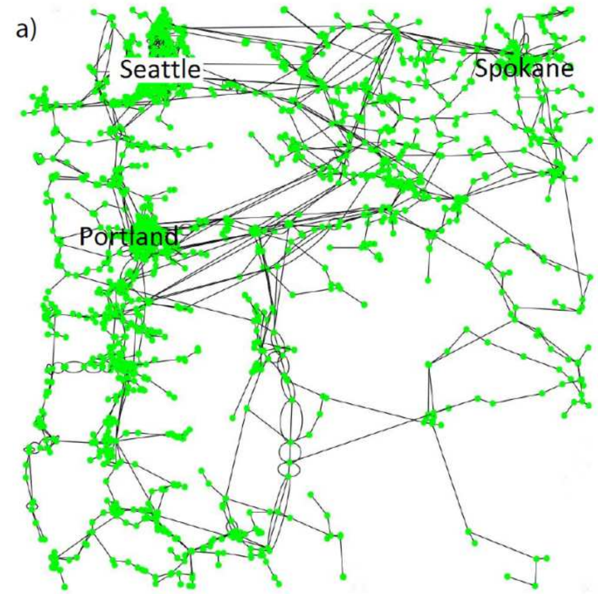
http://public.lanl.gov/rbent/SmarterGrids/

Part 1: Existence a Feasible Steady State in a Grid Subject to Failures—the "N-1 Problem"

<u>Deterministic generation—"N-1" security</u>

1. Remove a major component (~30,000)
2. Compute new power flow solution (~ 1 msec)
3. Verify there are no overloaded lines or generators or voltages out of bounds
4. Estimate of total time ~ 30 seconds
5. Embarrassingly parallel problem



<u>Deterministic generation—"N-2" security</u>

1. Remove two major components (~30,000 X 30,000)
2. 30 seconds → 250 hours
3. Even embarrassingly parallel won't help

<u>"N-1" works and "N-2" is typically not required</u>…..because component failures are rare and the probability of two occurring ("N-2") with sufficient spatiotemporal correlation to interact and cause problems is exceedingly rare…..assuming failures are independent….

Existence of a Feasible Steady State—Add 100 Wind/Stochastic Generators to the Grid—the "N-a Problem"

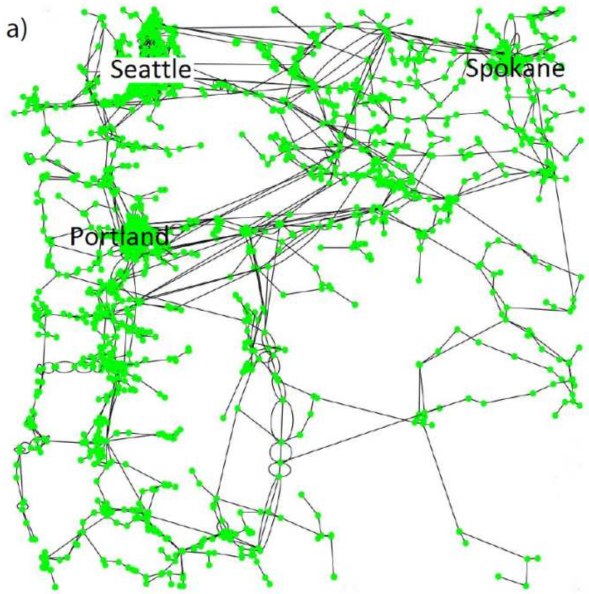
<u>**Concept #1—"N-1" checks for each wind generator**</u>

1. **"Fail" wind generators one at a time**
2. **Only adds 100 cases to the ~30,000**

<u>**Why doesn't this work?**</u>

1. **Wind generator "failure" is far more probable than component failure**
2. **Multiple generators will "fail" at the same time→Need to check (all possible configurations of failure) X (all N-1 cases).**
   $\sim 2^{100}$ X 30,000 ~ 10^{24} years
3. **And it is worse that this... ramps from ½ output to full output can be just as problematic**

a)



<u>**How many states do we assign to each stochastic generator?**</u> **3, 4….10?**

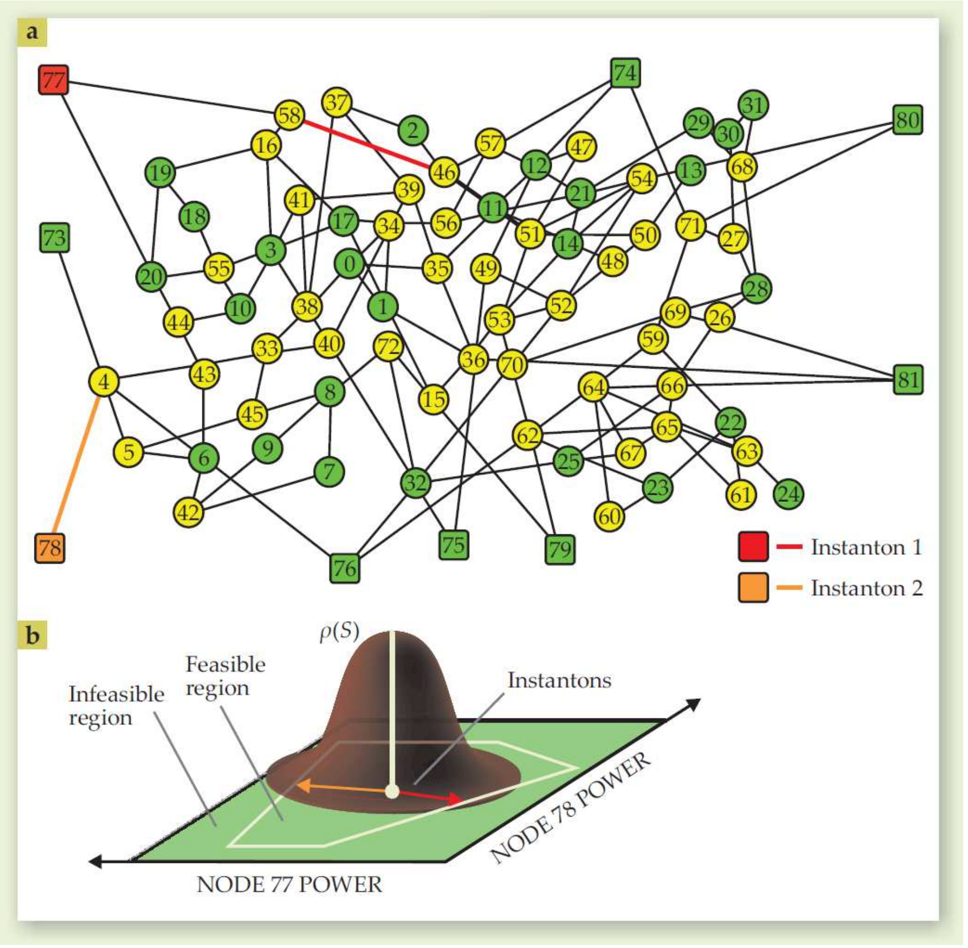
This approach does not scale……

Better search methods are needed to find the most probably (yet rare) modes of failure

Existence of a Feasible Steady State With Stochastic Generation—A Fundamentally New Approach Is Needed

<u>One Illustrative Example</u>—that leverages extreme statistics methods from statistical mechanics

1. **Consider full probability distributions over stochastic generation**
2. **Form the feasible space by taking the intersection of all *N* network constraints—turns out to be a convex region**
3. **Find the locations of highest probability of the stochastic generation on the boundary of the feasible region.**
4. **If feasible region is convex…. #3 is found by solving N linear programs.**
5. **Computational complexity reduced from say 2^{100} power flows to 100 LP's**



Probabilities of Infeasible Steady State Can Be Built Into an OPF—Chance Constrained OPF

Demonstrated computational tractability on a large network— ~4000 lines and ~400 generators—Polish grid from Matpower

R. Bent, D. Bienstock, and M. Chertkov, Synchronization-Aware and Algorithm-Efficient Chance Constrained Optimal Power Flow, proceedings of 2013 IREP Symposium-Bulk Power System Dynamics and Control, arxiv:1306.2972

$$\min_{p,\alpha} \mathbb{E}_{\boldsymbol{w}} \left[f(p - (e^T \boldsymbol{w})\alpha) \right]$$

$$\text{s.t.} \sum_{i \in \mathrm{G}} \alpha_i = 1, \quad \alpha \geq 0, \quad p \geq 0$$

$$\text{Prob}\left(\text{PF Eqs. are not feasible}\right) < \varepsilon$$

$$\text{Prob}\left(\beta_{ij} |\sin(\theta_i - \theta_j)| > \bar{p}_{ij}\right) < \epsilon_{ij} \quad \forall_{i,j \in \mathcal{E}}$$

$$\text{Prob}\left(p_g - (e^T \boldsymbol{w})\alpha_i > p_g^{max}\right) < \epsilon_g \quad \forall_{g \in \mathrm{G}}$$

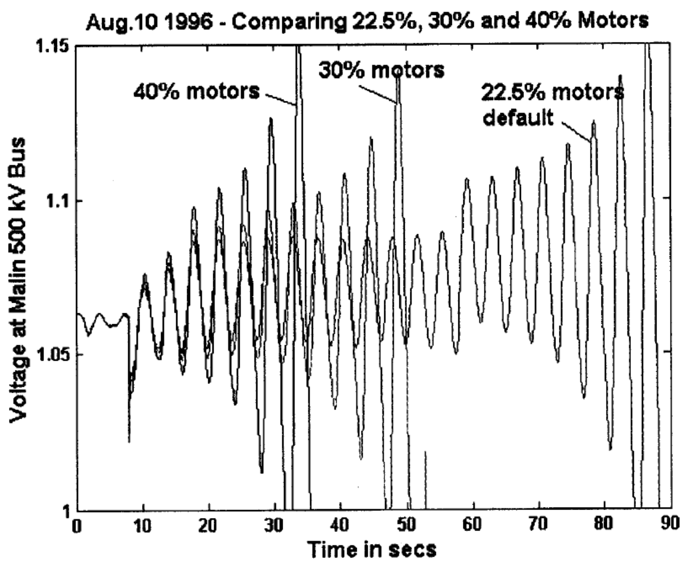
$$\text{Prob}\left(p_g - (e^T \boldsymbol{w})\alpha_i < p_g^{min}\right) < \epsilon_g \quad \forall_{g \in \mathrm{G}}$$

Numerical libraries needed to enable this family of approaches: ????

Part 2: Linear and Nonlinear Dynamical Models of Aggregated Distribution Loads (and Generators…)

Linear models of aggregated load

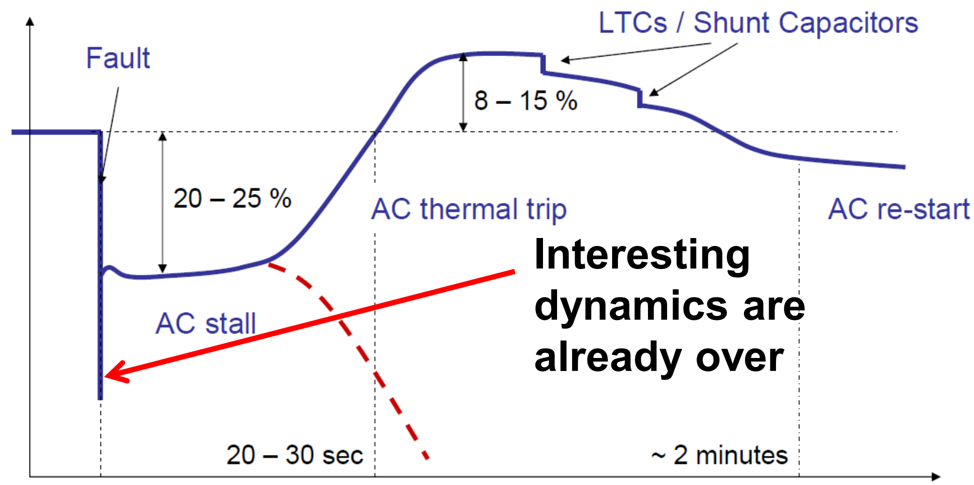
Uncertainty/Inaccuracy of linear dynamical load models creating uncertainty in estimates of small signal stability.



Pereira, L.; Kosterev, D.N.; Mackin, P.; Davies, D.; Undrill, J.; Wenchun Zhu, "An interim dynamic induction motor model for stability studies in the WSCC," *Power Engineering Society General Meeting, 2003, IEEE* , vol.3, no., pp.1404,1404, 13-17 July 2003

Nonlinear models of aggregated load

Close in fault causes induction motor stalling…subsequent large reactive currents keep voltage depressed and motor stalled.



Fault-Induced Delayed Voltage Recovery

Nonlinear Dynamical Models of Aggregated Distribution Loads

Power flow equations

$$\partial_z \rho = -p - r\frac{\rho^2+\phi^2}{v^2},$$

$$\partial_z \phi = -q - x\frac{\rho^2+\phi^2}{v^2},$$

$$\partial_z v = -\frac{r\rho + x\phi}{v}.$$

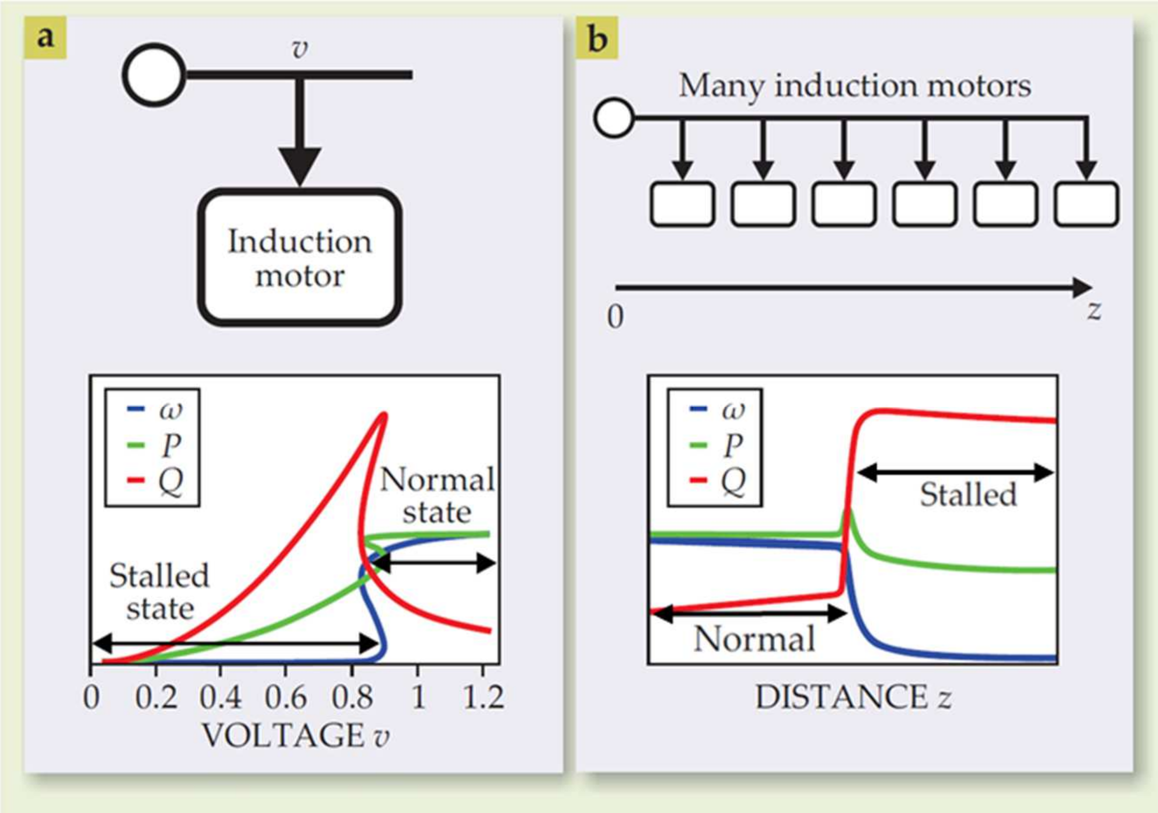
Motor dynamics

$$\mu \frac{d}{dt}\omega = \frac{p}{\omega_0} - t_0\left(\frac{\omega}{\omega_0}\right)^\alpha,$$

$$p = \frac{s r_m v^2}{r_m^2 + s^2 x_m^2},$$

$$q = \frac{s^2 x_m}{r_m^2 + s^2 x_m^2} v^2.$$

Coupling of power flow and motor dynamics creates a PDE that demonstrates collective behavior/collapse



Duclut, Charlie and Backhaus, Scott and Chertkov, Michael. Hysteresis, phase transitions, and dangerous transients in electrical power distribution systems. Phys. Rev. E **87**, 2013

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1. PDEs provide a good description of the dynamics
2. Adding a PDE at each feeder immediately leads to computational intractability
3. Need to reduce the PDE to several ODE's at each feeder to even come close to a tractable problem
4. Model reduction of the nonlinear behavior is difficult

Numerical library needs—generalized linear and nonlinear equivalent models